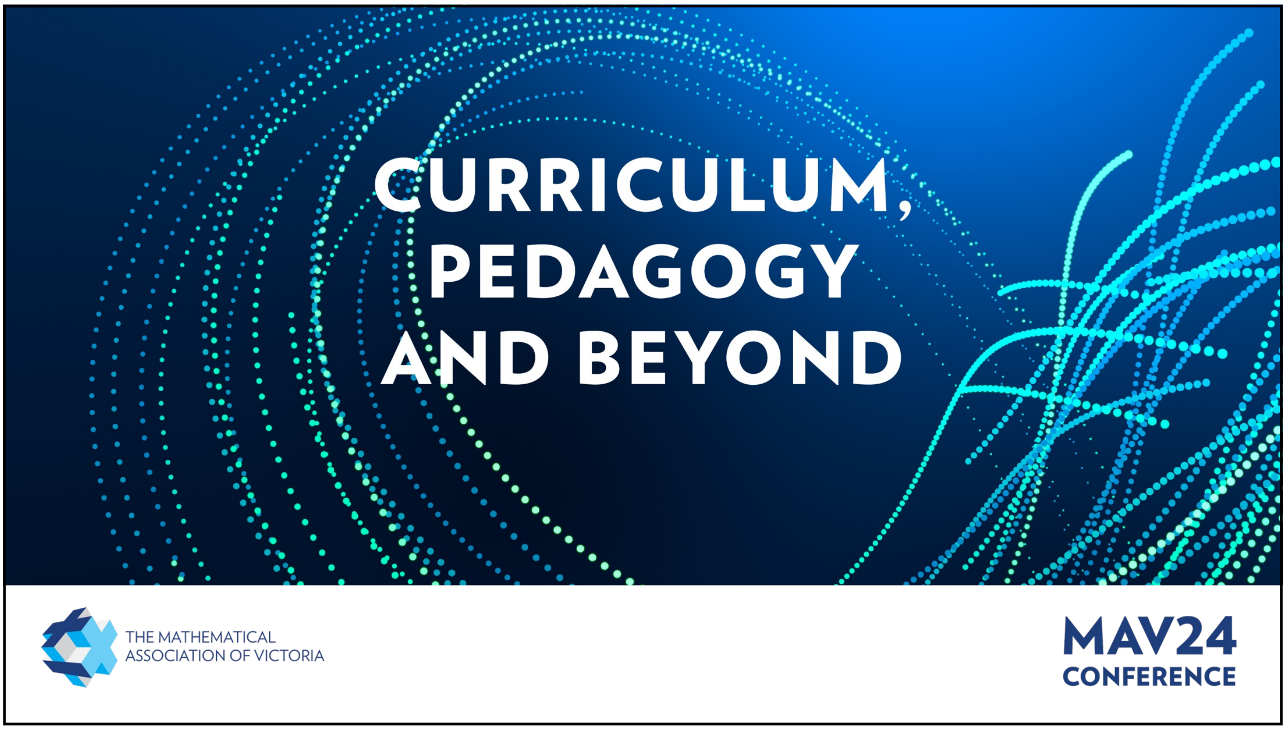
CURRICULUM,
PEDAGOGY
AND BEYOND
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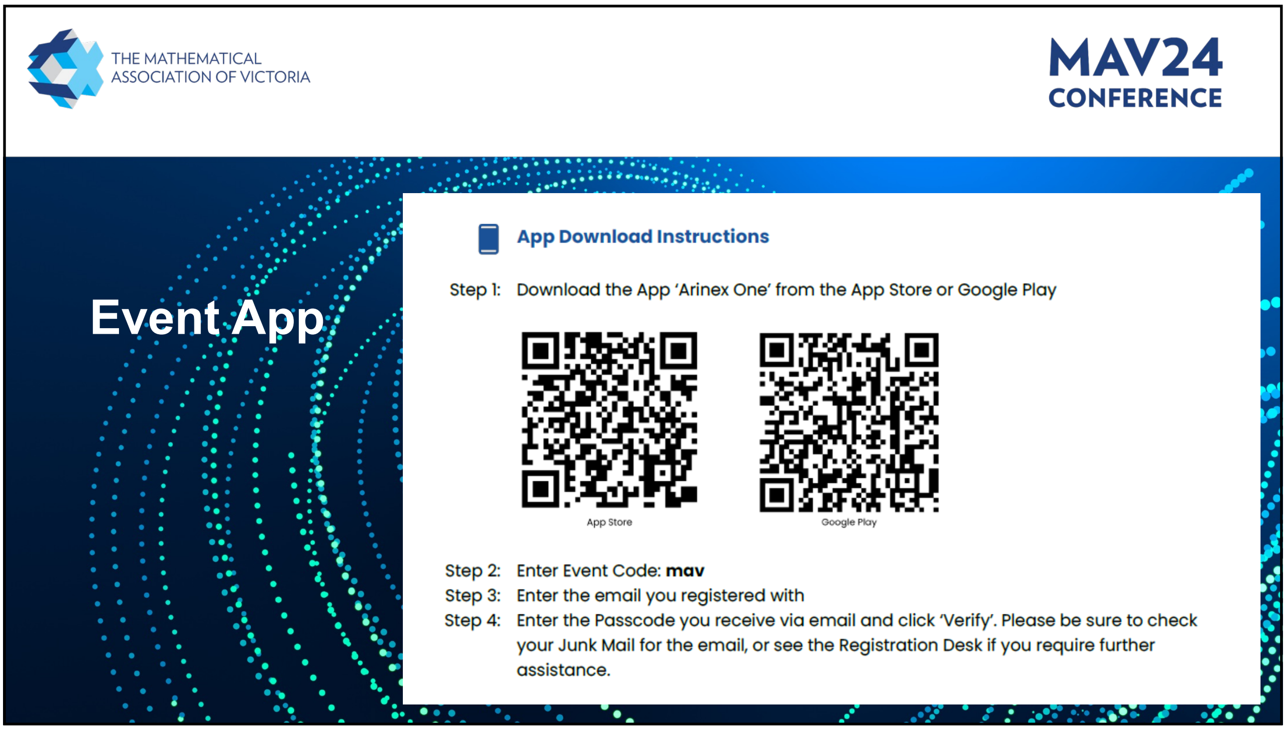
1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Event App
App Download Instructions
Step 1: Download the App 'Arinex One' from the App Store or Google Play
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Google Play
Step 2: Enter Event Code: mav
Step 3: Enter the email you registered with
Step 4: Enter the Passcode you receive via email and click 'Verify'. Please be sure to check your Junk Mail for the email, or see the Registration Desk if you require further assistance.

2

THE MATHEMATICAL ASSOCIATION OF VICTORIA

MAV24 CONFERENCE

The BIG IDEAS in NUMBER Update

Emeritus Professor Dianne Siemon

3

Crowded Curriculum

Differentiating Everything

Teachers are working too hard on things that do not necessarily progress students' mathematics learning

Marking Homework

Ability Grouping

Data Walls

Frequent Pre-tests & Post-tests

Over Scaffolding

Rotation Groups

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4

An unrelenting focus on learning

Requires we recognise and act on three key processes in learning.

An understanding of:

- **where the learner is right now;**
- **where the learner needs to be; and**
- **how to get there.**

Wiliam, D. (2013). Assessment: The bridge between teaching and learning. *Voices from the Middle, 21*(2), 15 – 20

But **where** in relation to what?

Year level curriculum expectations or what research suggests is most likely to make a difference?

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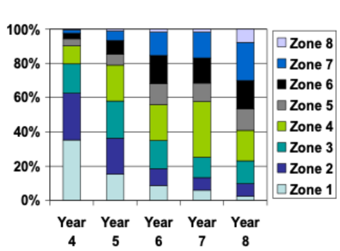
5

What can we learn from research?

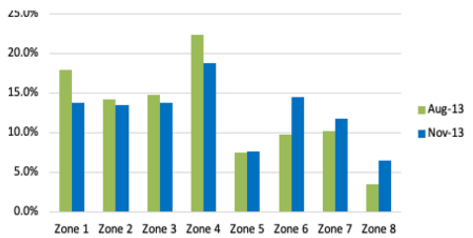
Access to multiplicative thinking largely explains the seven-year range in student mathematics achievement in the middle years.

Identifying and responding to student's learning needs in relation to multiplicative thinking leads to substantive improvements in student's mathematics achievement (0.4 ≤ ES ≤ 1.9).

(Siemon & Virgona, 2021; Siemon et al., 2006, 2018; Siemon, 2019)



SNMY (N=3200, ICSEA > 1000)
Average Effect Size* = 0.5



RMF-P (N=1792, ICSEA < 1000)
Average Effect Size = 0.64

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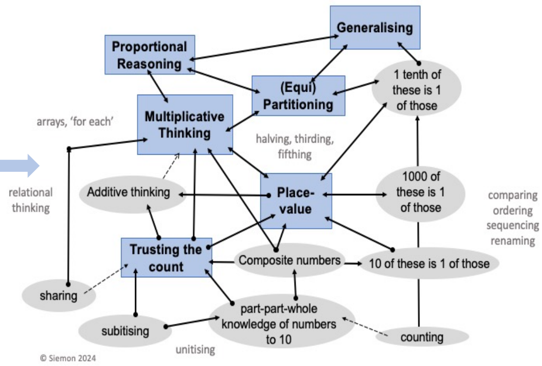
The Origin of the BIG IDEAS

LAF Zone	Level Description
8	Solves and justifies a wide range of problems involving unfamiliar multiplicative situations including fractions and decimals, solves complex proportional reasoning problems, formally describe patterns in terms of general rules, solves complex, open-ended problems
7	Compare, order, sequence, represent, and rename whole numbers, fractions, decimals, integers, inverse and identity relations, structure of place value system, recognise, describe and apply relationships between variables, algebraic processes, ratio, complex proportion problems
6	Extend decimal fractions, use partitioning strategies, more efficient, processes for dealing with all four operations, **proportion** problems, notion of variable, formally describe patterns
5	Uses **partitioning** strategies locate and re[illegible] value larger whole numbers and tenths, fle[illegible] multiplying and dividing, area idea, Cartesi[illegible] and multiples, strategies for adding and su[illegible]
4	More efficient strategies for multiplying and [illegible] numbers, Tenths as a new place-value par[illegible] compare fractions, times as many idea
3	**Place-value** based strategies, simple proportion problems, Cartesian product, thirding and fifthing partitioning strategies, key fraction generalisations, works with simple patterns
2	More efficient strategies for counting large collections, array/region-based strategies for multiplication facts, halving partitioning strategies to create fraction representations, key fraction generalisations, extended place value
1	**Trust the count** (part-part-whole knowledge), mental strategies for addition and subtraction, 2 and 3-digit place v[illegible] arrays and regions

Item analysis pointed to a small number of **BIG IDEAS** in Number that needed to be developed and consolidated over time

The Learning Assessment Framework for Multiplicative Thinking (SNMY 2004-2006)

Assessment for Common Misunderstandings F-10 (AfCM, 2006)



The BIG IDEAS underpinning Multiplicative Thinking

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What is a 'Big Idea' in this context?

- An idea, strategy, or way of thinking about some key aspect of mathematics, **without which students' progress in mathematics will be seriously impacted** (e.g., trusting the count, place value)
- Encompasses and **connects many other ideas and strategies** (e.g., multiplicative thinking)
- Provides an **organising structure** or a frame of reference that supports further learning and generalisations (e.g., place value)
- Cannot be completely defined but **can be observed in activity** (e.g., partitioning, proportional reasoning, generalising).

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Need to be in place by the end of …

First 18 months	**Trusting the count** - developing flexible mental objects for the numbers 0 to 10, part-part-whole knowledge
Year 2	**Place-value** - the importance of moving beyond counting by ones, the structure of the base ten numeration system
Year 4	**Multiplicative thinking** (initial ideas) - the key to understanding rational number and developing efficient mental and written computation strategies in later years
Year 6	**Partitioning** (equal parts) - the missing link in building common fraction and decimal knowledge and confidence
Year 8	**Proportional reasoning** - extending what is known beyond rule-based procedures to represent and solve problems involving fractions, decimals, percent, ratio, rate and proportion
Year 10	**Generalising/Formalising** - skills and strategies to support equivalence, recognition of number properties and patterns, and the use of algebraic text

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Everyone's Responsibility …

	F-1	1-2	3-4	5-6	7-8
Trusting the Count	Mental objects to 10	Mental strategies	Number facts	Extended number facts	Efficient estimation
Place Value	1 ten and…	10 of these is 1 of those	1 tenth of these is 1 of those	1 thousand of these is 1 of those	Structure of base 10
Multiplicative Thinking	Composite unit, doubles to 20, sharing	Arrays & regions, doubling, quotition & partition	For each & area idea Mental strategies	Factor-factor-product idea, efficient strategies	Rate, ratio, percent, extended strategies
Partitioning	Mental objects to 10, sharing	Halving& thirding strategies	Fifthing and tenthing strategies	Renaming fractions	Rate, ratio, percent
Proportional reasoning	Many to one counts, simple rate (for each) problems	Locating numbers on a number line, horizon problems	For each idea, mental strategies,	Factor-factor-product idea, scale	Rate, ratio, percent, missing value problems

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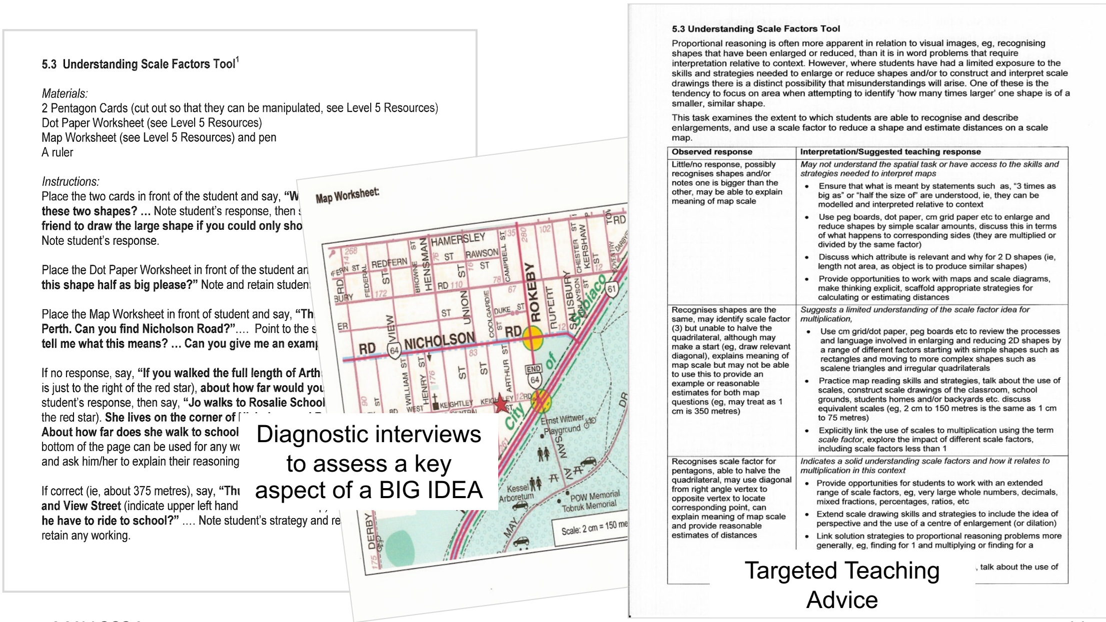
The original AfCM Resources



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What's new in AfCM+?

- Considerably expanded and updated User's Guide
- Each BIG IDEA linked to the new Curriculum
- Updated description of Common Misunderstandings
- Revised and additional Tools
- Sample recording forms for each Tool
- Expanded and updated Teaching Advice
- Suggestions for use with small groups or the whole class

Largest? Smallest? Why?

87 tens 17 ones

7 hundreds 43 ones

88 tens 5 ones

693 ones

3 hundreds and 52 ones

36 tens and 4 ones

Additional tasks from Tools 2.3 and 2.4

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AfCM+ User's Guide

- Purpose
- Intended Use → assessment FOR/AS learning
- Everyone's Responsibility
- Accessing AfCM
- Choosing a Tool
- Materials
- Administration → not limited to one-on-one interviews
- Teaching Response
- Targeted Teaching → not ability grouping!
- References

AfCM+ User's Guide

Purpose - To provide F-10 teachers with the means to identify and respond to student learning needs in relation to key aspects of the Big Ideas in Number without which their progress in school mathematics will be seriously impacted.

Intended Use – The Tools and the associated Teaching Advice are particularly useful in identifying and responding to the learning needs of students who teachers believe are 'at risk' or likely to be at risk in relation to these important underpinnings. However, they can also be used to obtain more accurate or in-depth information about students who teachers feel are under-achieving and provide direction to teachers looking to extend particular students.

Everyone's responsibility – an important learning from the many schools that have used the Tools and a recent review of the literature on Big Ideas (Siemon, 2022a) is the importance of understanding the role that the Big Ideas in Number play in subsequent year Levels. While the Tools are aimed at identifying student learning needs in relation to the key, underpinning aspects of each Big Idea, the sequence of Big Ideas serve another purpose (see Figure 1). All teachers at all levels need to be able to 'look back' to appreciate the role that these underpinning ideas and strategies play in supporting further learning. Equally, all teachers need to be able to 'look forward' to understand where the knowledge is going and what these underpinning ideas support. The first of these two responsibilities is about understanding the pre-requisite knowledge and skills needed to engage successfully with the curriculum at a particular level. The second is what Ball (1993) refers to as *horizon knowledge*, a key aspect of the knowledge of mathematics needed for teaching.

Figure 1. Big Ideas in Number – the responsibility of all

	F	1-2	3-4	5-6	7-8
Trusting the Count	Mental objects to 10	Mental strategies	Number facts	Extended number facts	Efficient estimation
Place Value	1 ten and…	10 of these is 1 of those	1 tenth of these is 1 of those	1 thousand of these is 1 of those	Structure of base 10
Multiplicative Thinking	Composite unit, doubles to 20, sharing	Arrays & regions, doubling, quotition & partition	For each & area idea Mental strategies	Factor-factor-product idea, efficient strategies	Rate, ratio, percent, extended strategies
Partitioning	Mental objects to 10, sharing	Halving& thirding strategies	Fifthing and tenthing strategies	Renaming fractions	Rate, ratio, percent
Proportional reasoning	Many to one counts, simple rate (for each) problems	Locating numbers on a number line, horizon problems	For each idea, mental strategies,	Factor-factor-product idea, scale	Rate, ratio, percent, missing value problems
Generalising/ Formalising	At all levels and across all strands of the mathematics curriculum, students need regular opportunities to form and test conjectures, identify patterns, and generalise from multiple instances to support algebraic reasoning in later years				

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BIG IDEAS Linked to Curriculum

AFCM+ Focusing on the Big Ideas in Number
PLACE VALUE

Relevant Achievement Standards

By the end of Year 2, students order and represent numbers to at least 1000, apply knowledge of place value to partition, rearrange and rename two- and three-digit numbers in terms of their parts, and regroup partitioned numbers to assist in calculations. … Students recall and demonstrate proficiency with addition and subtraction facts within 20 and multiplication facts for twos.

- **VC2M2N01**: recognise, represent, and order numbers to at least 1000 using physical and virtual materials, numerals, and number lines (AC9M2N01)
- **VC2M2N02**: partition, rearrange, regroup, and rename two- and three-digit numbers using standard and non-standard groupings; recognise the role of a zero digit in place value notation (AC9M2N02)
- **VC2M2N04**: add and subtract one- and two-digit numbers, represent problems using number sentences and solve using part-part-whole reasoning and a variety of calculation strategies (AC9M2N04)

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Trusting the Count

Is evident when children:

- know that **counting is an appropriate response** to questions which ask how many;
- believe that counting the same collection again will always produce the **same result** irrespective of how the objects in the collection are changed or manipulated (WA First Steps);
- recognise collections to 10 without counting (i.e. *subitise*);
- have **access to mental objects** for each of the numbers to ten which they can use without having to represent, count or see these collections physically (i.e. *part-part-whole knowledge*);
- demonstrate a **sense of numbers beyond ten**; and
- **use small collections as units** when counting larger collections (Siemon, 2005).

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Sharing Tool (AfCM+, Tool 1.3)

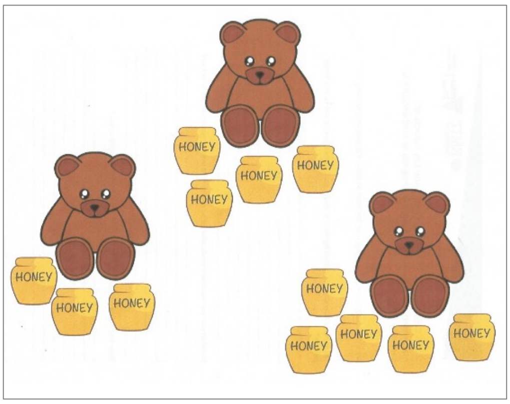
This new Tool examines the extent to which students understand sharing and the importance of fair shares (equal groups). …

Sharing relates to *Trusting the Count* as it indicates the extent to which students can recognise the shares as *composite units* … and make judgements about group size without counting

This Tool also explores *part-part-whole* idea where parts are equal

Materials:

- 24 counters
- 4 paper plates
- 20-24 cm paper streamer or string
- 1 small Kinder Square (otherwise known as Brennix paper)
- Sharing Card



Is this a fair share? What would you do to make sure each bear gets the same amount of honey?

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Place Value (Whole Number)

Is evident when children:

- know that **10 of these is 1 of those** (e.g., by making and naming numbers to 1000 and by playing trading games);
- **make, name and record** numbers to 1000 and beyond using appropriate materials;
- **compare and order** whole numbers to 1000;
- **count forwards and backwards in place value parts** well beyond 100 (e.g., 87, 97, 107, 117, 127, …);
- **locate whole numbers to 1000 on a number line** explaining or justifying their decision based on benchmarks (e.g., it's about half);
- **rename numbers in terms of their place value parts** (e.g., recognise that 367 can be represented and renamed as 36 tens and 7 ones or 367 ones).

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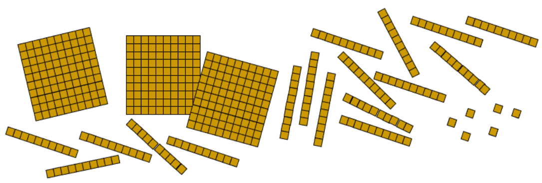
Renaming and Calculating Tool (AfCM+, Tool 2.4)

This revised Tool indicates the extent to which students can rename three-digit numbers and have access to place-value based strategies to support efficient calculation (e.g., works with 3 tens rather than 30 ones).



Write the number shown (476). If you could only use tens to make this number, how many would you need?

There are 3 hundreds and 52 ones here (left) and 36 tens and 4 ones here (right), which pile shows the largest number?

713

Say or write the number that is 4 tens larger than this number?

5308

Say or write the number that is 4 hundreds less than this number?

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Multiplicative Thinking – A Really BIG IDEA

Multiplicative thinking involves recognising and working with relationships between different quantities and with processes such as enlarging or shrinking a given quantity (Siemon, 2022).

Multiplicative thinking is qualitatively different to additive thinking. It is evident when students:

- work flexibly and confidently with an extended range of numbers (i.e., larger whole numbers, fractions decimals, per cent, and ratios);
- solve problems involving multiplication and division (i.e., some form of proportion) using strategies appropriate to the task; and
- explain and communicate their reasoning in a variety of ways (e.g., words, diagrams, symbolic expressions, and written algorithms. (Siemon, Breed, & Virgona, 2005).

Droplets of moisture from a sneeze have been measured travelling at 165 km/hour. How many cm/sec is this?

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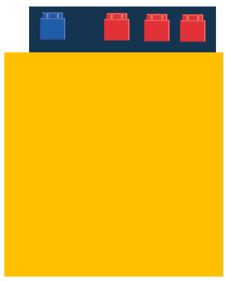
Composite Units Tool (AfCM+, Tool 3.1)

This Tool replaces the original Tool 3.1. It assesses the extent to which students can work with small numbers as *abstract composite units* (i.e., as countable units in the absence of physical materials/models).

The idea that a count can be counted is a difficult notion for some students, but it is an essential underpinning for place-value, multiplication, and division. This task should only be used where students have demonstrated some capacity to physically count collections by twos, fives and tens (see Tool 2.2) and some experience working with threes.



Materials:

- a collection of blue and red unifix blocks or similar
- For Each Cards
- A non-transparent sheet of paper/card to completely cover the first For Each Card
- Balloon Problem Card

Cover all but first row. Say: For each blue block there are 3 red block. For 5 blue blocks, how many red blocks will there be?

If difficult, provide at least 8 blue and 24 red unifix blocks and say: Can you use these to show how many red blocks there will be be for 5 blue blocks?

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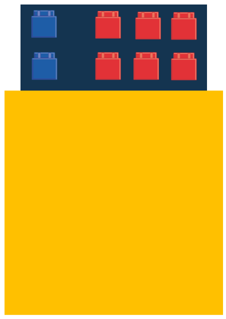
Composite Units Tool (AfCM+, Tool 3.1) cont.



If the initial task completed easily (15 red blocks), show two rows and ask: For 8 blue blocks, how many red blocks will there be?

If completed easily ask: If there were 42 red blocks, how many blue blocks will there be?

If this task completed, say: For each orange block there are two blue blocks. …Remember for each blue block there are three red blocks… For two orange blocks, how many red blocks will there be? …

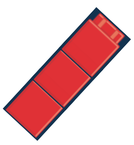
How much would 20 balloons cost?

Times as Many Tool (AfCM+, Tool 3.5)

This is a new Tool that identifies the extent to which students understand the *times as many* or *times as much* idea for multiplication and division. … This idea is important as it supports multiplication and division by non-integral amounts and the more general notion of a factor which is needed to support algebra.

Materials:

- Unifix blocks (or similar)
- Fast Cards
- Pen/pencil and paper
- Recipe Card



This is a 3-rod, can you make one that is 7 times as long?

If correct (21), connect to 3-rod and say: Can you make another rod that is 1 quarter as long as this rod.

We walk at 4 kilometres/hour.

I can fly 30 times faster.

How fast can I fly?

We walk at 4 kilometres/hour.

A plane can fly 200 times faster. How fast does the plane fly?

16 metres, how many centimetres?

78 centimetres, how many millimetres?

A spaghetti recipe for 4 people needs 360 grams of pasta. To make this recipe for 6 people, how many grams of pasta are needed?

(equi)Partitioning

Is evident when students:

- use **halving strategies to locate fractions** in the halving family on a number line;
- **estimate thirds and fifths by building on what is known** (e.g., a third is smaller than a half … a fifth is smaller than a quarter);
- construct and **use their own fraction models to compare, order and rename fractions**;
- use **partitioning strategies to locate decimal fractions** on an open number line based on benchmarks (e.g., it's about half);
- recognise the **role of factors in renaming common fractions** and draw on their knowledge of place value **to rename decimal fractions and measures in multiple ways;**
- **think of division in terms of 'what do I multiply by?'** (i.e., factors).

4 tenths

27 hundredths

27 tenths

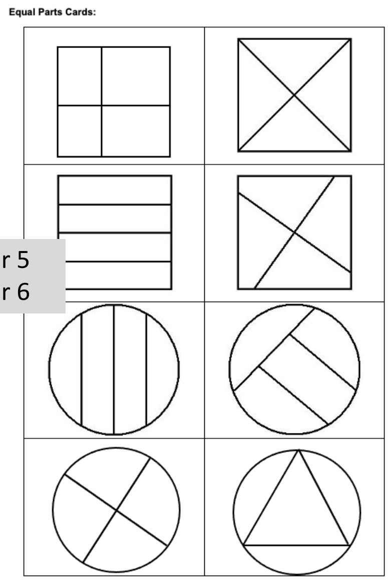
705 thousandths

247 hundredths

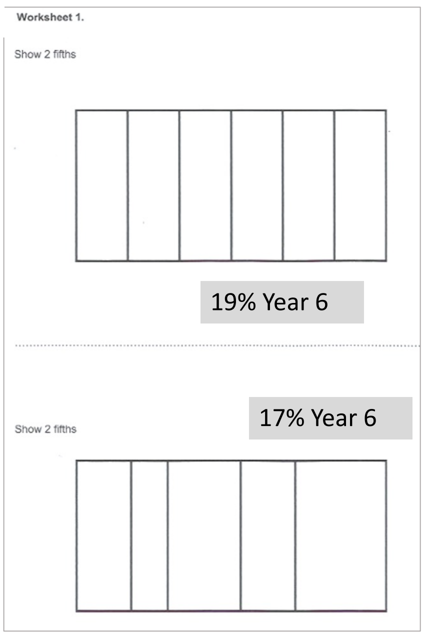
42 thousandths

From Tool 4.6 Comparison and Ordering Tool

Equal Parts Tool (AfCM+ Tool 4.1)



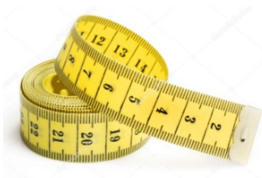
17% Year 5
22% Year 6



19% Year 6

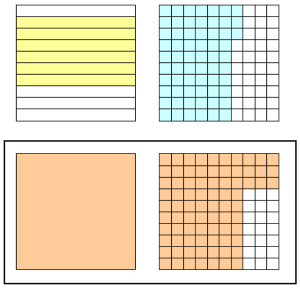
17% Year 6

Decimal Fraction Naming & Recording Tool (AfCM+, Tool 4.5)



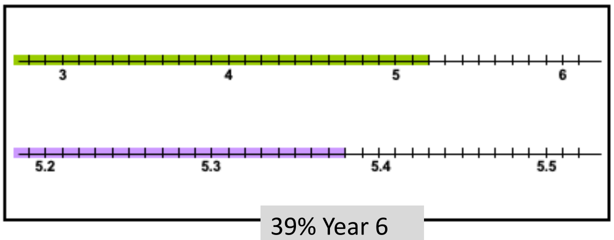
Measure and record (145 cm or 1.45 m)

What's this got to do with what you have there? …



Write shaded fraction. Is there another way to write that? …

What is the length of the line?



39% Year 6

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Comparing and Ordering Tool (AfCM+, Tool 4.6)

$\frac{2}{3}$ $\frac{48}{80}$

25% Year 6

Which is smaller? Why?

3.21	3.012
3.201	2.312
2.32	3.3

13% Year 6

9% Year 6

Put these in order from smallest to largest

- 4 tenths
- 27 hundredths
- 27 tenths
- 705 thousandths
- 247 hundredths
- 42 thousandths

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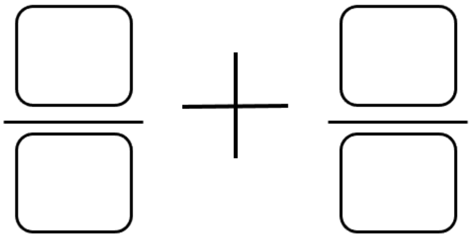
An adaptation of Tool 4.6

In small mixed ability groups use the cards below to make two fractions that when added together are as close as possible to one.



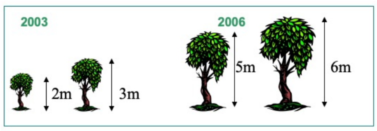
1 3 4 5 7

Justify your choice

Proportional Reasoning

Is evident when students:

- notice and use **multiplicative relationships** between quantities;
- **work confidently with intensive quantities** (e.g., kilometres/hour, mass/unit volume);
- recognise and **solve comparison problems** involving quantitative relationships (e.g., feral cat problem);
- recognise and **solve missing value problems** (e.g., beanie problem); and
- use a **scale factor** to reduce or enlarge diagrams and find a distance on a map.



Which tree grew more?

Samantha's Snail travelled 1.59 metres in 6 minutes. Jeanie's Snail took 4 minutes and 30 seconds to travel 126 cm. How long did each snail travel in 9 minutes?

Thuan lives on the corner of Redfern St and View St, about how far does he have to ride to go to school?

Updated Teaching Advice

What the tool is assessing and why it is important

Observed responses

Interpretation of what each response means

Teaching advice (dot points)

5.3 Understanding Scale Factors Tool

Proportional reasoning is often more apparent in tasks involving visual images (e.g., recognising shapes that have been enlarged or reduced or reading scale diagrams), than in word problems that require interpretation relative to context. However, where students have had a limited exposure to the skills and strategies needed to enlarge or reduce shapes and/or to construct and interpret scale drawings there is a distinct possibility that misunderstandings will arise. One of these is the tendency to focus on area when attempting to identify 'how many times larger' one shape is of a smaller, similar shape.

This task examines the extent to which students are able to recognise and describe enlargements and use a scale factor to reduce a shape and estimate distances on a scale map.

Observed response	Interpretation/Suggested teaching response
Little/no response, possibly recognises shapes and/or notes one is bigger than the other, may be able to explain meaning of map scale	*May not understand the spatial task or have access to the skills and strategies needed to interpret maps* • Ensure that what is meant by statements such as, "3 times as big as" or "half the size of" are understood (i.e., they can be modelled and interpreted relative to context) • Use peg boards, dot paper, cm grid paper etc to enlarge and reduce shapes by simple scalar amounts, discuss this in terms of what happens to corresponding sides (they are multiplied or divided by the same factor) • Discuss which attribute is relevant and why for 2 D shapes (i.e., length not area, as object is to produce similar shapes) • Provide opportunities to work with maps and scale diagrams, make thinking explicit, scaffold appropriate strategies for calculating or estimating distances
Recognises shapes are the same, may identify scale factor (3) but unable to halve the quadrilateral, although may make a start (e.g., draw relevant diagonal), explains meaning of map scale but may not be able to use this to provide an example or reasonable estimates for both map questions (e.g., may treat as 1 cm is 350 metres)	*Suggests a limited understanding of the scale factor idea for multiplication,* • Use cm grid/dot paper, peg boards etc to review the processes and language involved in enlarging and reducing 2D shapes by a range of different factors starting with simple shapes such as rectangles and moving to more complex shapes such as scalene triangles and irregular quadrilaterals • Practice map reading skills and strategies, talk about the use of scales, construct scale drawings of the classroom, school grounds, students homes and/or backyards etc. discuss equivalent scales (e.g., 2 cm to 150 metres is the same as 1 cm to 75 metres) • Explicitly link the use of scales to multiplication using the term *scale factor*, explore the impact of different scale factors, including scale factors less than 1
Recognises scale factor for pentagons, able to halve the quadrilateral, may use diagonal from right angle vertex to opposite vertex to locate corresponding point, can explain meaning of map scale and provide reasonable	*Indicates a solid understanding scale factors and how it relates to multiplication in this context* • Provide opportunities for students to work with an extended range of scale factors (e.g., very large whole numbers, decimals, mixed fractions, percentages, ratios, etc) • Extend scale drawing skills and strategies to include the idea of perspective and the use of a centre of enlargement (or

From the *Assessment for Common Misunderstandings + Materials* (Siemon, 2024)

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Targeted Teaching

Targeted teaching is a form of differentiation that is focused on addressing students' specific learning needs in relation to a small number of really 'big ideas' in Number, **without which students' progress in school mathematics will be seriously impacted**.

Siemon, 206, 2017, 2024)

Take Away: Not everything needs to be differentiated

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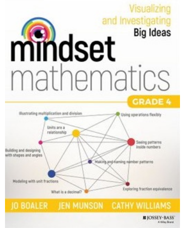
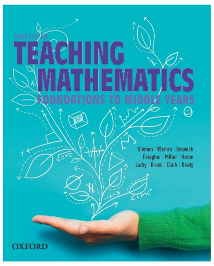
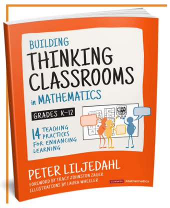
Note: Targeted Teaching ≠ Ability Grouping

- Students need to work in mixed ability groups on rich, accessible, but challenging tasks for the majority of maths time (Boaler, 2008; Sullivan, 2011)
- Flexibility and student choice are key







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https://www.education.vic.gov.au/Documents/school/teachers/teachingresources/discipline/maths/teaching-with-the-big-ideas-in-mathematics.pdf

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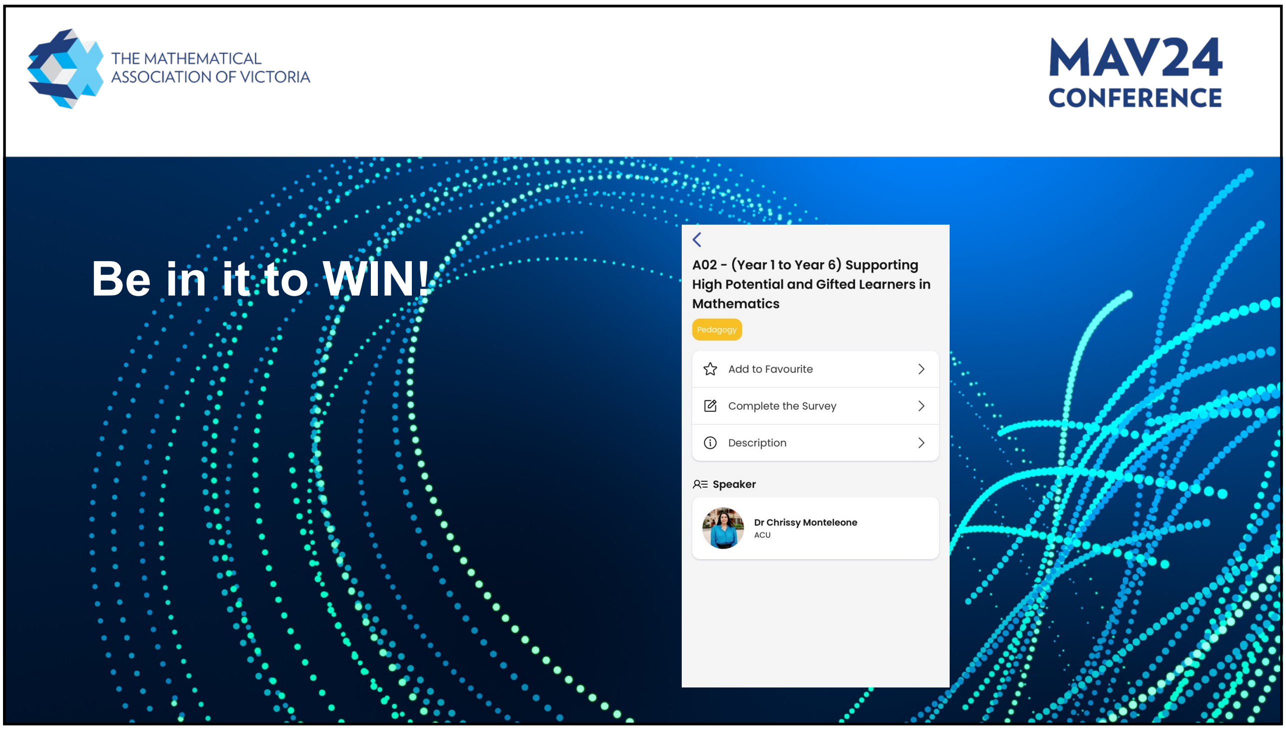
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